

# Scale-Invariant Fluctuations from Galilean Genesis

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We study the spectrum of cosmological fluctuations in scenarios such as Galilean Genesis [1] in which a spectator scalar field acquires a scale-invariant spectrum of perturbations during an early phase which asymptotes in the far past to Minkowski space-time. In the case of minimal coupling to gravity and standard scalar field Lagrangian, the induced curvature fluctuations depend quadratically on the spectator field and are hence non-scale-invariant and highly non-Gaussian. We show that if higher dimensional operators (the same operators that lead to the  $\eta$ -problem for inflation) are considered, a linear coupling between background and spectator field fluctuations is induced which leads to scale-invariant and Gaussian curvature fluctuations.

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## I. INTRODUCTION

Inflation [2] is currently the most well-studied and successful paradigm for very early universe cosmology. Most importantly, inflation is the first scenario based on causal physics to give a mechanism [3] to generate perturbations on the scales currently observed. The predicted spectrum is nearly scale-invariant and close to Gaussian. These predictions have been spectacularly confirmed in recent cosmic microwave (CMB) measurements [4].

As already pointed out [5] a decade before the development of inflationary cosmology, any early universe scenario which yields an almost adiabatic and nearly scale-invariant spectrum of fluctuations on scales which are super-Hubble at late times (e.g. the time of equal matter and radiation) will yield an angular power spectrum of CMB anisotropies in agreement with observations. Inflation is the first but not the only model which yields such fluctuations.

It is valuable to investigate alternative scenarios to obtain a scale-invariant spectrum of curvature fluctuations using causal microphysics. On one hand, one does not know apriori whether inflation is indeed the right paradigm. At least in its current realizations based on coupling scalar fields to Einstein gravity it suffers from a number of conceptual challenges (see e.g. [6]), and if inflation in fact turns out not to be the correct paradigm, alternatives are certainly needed. On the other hand, even if inflation turns out to be the correct paradigm, competing scenarios are useful because they may provide guides to future experiments which will allow inflation to pass further non-trivial tests. For example, the “String Gas Cosmology” alternative to inflation [7] predicts a blue tilt of the spectrum of gravitational waves, whereas inflationary models developed within the context of Einstein gravity generically produce a small red tilt. Thus, future measurements of the slope of the spectrum of gravitational waves could either falsify inflation or allow it to pass a further non-trivial test. In this sense having alternatives to inflation makes early universe cosmology a healthier science.

Recently, there is an increasing interest in a class of alternatives to inflation in which the scale-invariance of the fluctuations is induced not by the de-Sitter-like expansion of space, but by the evolution of another matter field. Examples are the *conformal cosmology* of [8] (see also [9–11]), in which the evolution of a conformal scalar field induces scale-invariant fluctuations in an axion-like field to which it couples, the *pseudo-conformal cosmology* of [12] and the *Galilean genesis* model of [1] in which a Galileon field which dominates the background dynamics of space-time induces scale-invariant fluctuations of a spectator scalar fields.

In this paper we propose a mechanism by which the scale-invariance of the spectator scalar fields can be transferred to a scale-invariant spectrum of curvature fluctuations. The standard curvaton mechanism does not work since in this case the curvature fluctuations are quadratic in the spectator scalar field, and hence the fluctuations will be neither scale-invariant nor Gaussian. The key to our construction is to introduce a linear coupling between the nontrivial background geometry and the spectator scalar.

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## II. EMERGENT DYNAMICS FROM EMERGENT COSMOLOGY

The three scenarios mentioned above all can be viewed as particular realizations of an *emergent cosmology*, in the sense that there is a spectator field whose equation of motion looks like that of a standard scalar field in a de Sitter background geometry, while the actual space-time is not de Sitter at all. For example, in the *Galilean Genesis* scenario it is an evolving Galileon field which couples to a spectator scalar field like a de Sitter metric would, while the Galileon field leads to a metric which approaches that of flat Minkowski space-time as time tends to  $t \rightarrow -\infty$ . In this case, the expanding background expansion is in fact emergent from an initial static phase (as is also postulated to happen in String Gas Cosmology [7]).

There is, however, a generic problem which arises if one generates a scale-invariant spectrum of fluctuations of a spectator scalar field in the hope of then obtaining scale-invariant curvature perturbations [13]:

- If the scalar field has no potential term, the scale-invariant perturbation of the scalar field couples quadratically to the curvature perturbation  $\zeta$  as  $\dot{\zeta} \sim \delta\phi^2$ , and thus the resulting curvature perturbation is neither scale invariant nor Gaussian.
- If the spectator scalar field has a potential (say, a fixed mass term) and its background value is excited and evolving in time (like the inflation and the curvaton are in inflationary cosmology), then - although the background scalar field allows a linear coupling between the spectator field fluctuations and the curvature perturbation - the background spectator scalar field will destroy the emergent background phase in the far past, when fluctuations are supposed to be generated.

Actually, the solution to the above problems is built-in in those theories. To see this, it is helpful to pause and revisit the “ $\eta$ -problem” for inflation [15]. The  $\eta$ -problem states that in a time-dependent background, a massless field  $\phi$  typically gets a mass of order of the Hubble parameter, unless the masslessness is protected by a symmetry such as a shift symmetry [16]. For example, in the context of supergravity models, then if the field comes from a Kähler potential, the Lagrangian will have the structure

$$\mathcal{L} \supset V_0 \exp \left[ \mathcal{O}(1) \frac{\phi^2}{M_p^2} \right] , \quad (1)$$

where  $V_0$  is the effective vacuum energy in the system, and  $M_p$  is the Planck mass. Expanding this equation, the  $\phi$  field obtains a mass of order Hubble parameter  $H$ :

$$\mathcal{L} \supset \left[ 1 + \mathcal{O}(1) \frac{\phi^2}{M_p^2} \right] V_0 \simeq V_0 + \mathcal{O}(1) H^2 \phi^2 . \quad (2)$$

It is believed that the above effect is so general that it becomes a problem of inflation.

There is good reason to believe that the same term should also naturally arise in emergent universe scenarios where the Hubble parameter is emergent (specifically in the Galilean Genesis model). Thus, a light scalar field  $\phi$  (our spectator scalar field) obtains an emergent mass. The emergent mass, on one hand, breaks the shift symmetry. This will induce a linear coupling of the scalar field fluctuations to curvature perturbations, and will allow a scale-invariant spectrum of the spectator field to induce a scale-invariant spectrum of curvature fluctuations. On the other hand, the mass of the spectator field is emergent and thus the induced potential of the field does not destroy the initial emergent background. Specifically, it does not break the emergent nature of expansion of the universe.

For this purpose, starting with the Lagrangian  $\mathcal{L}_m$  for a massless scalar field, we shall consider the corrected matter Lagrangian

$$\mathcal{L}_m \rightarrow \left( 1 + \frac{\phi^2}{M^2} \right) \mathcal{L}_m , \quad (3)$$

where  $M$  is a mass parameter characteristic of the new physics which yields the corrections to the Lagrangian. This mass will induce non-trivial dynamics of  $\phi$  which in turn leads to a linear coupling between fluctuations in  $\phi$  and curvature perturbations.

Alternatively, we could also consider a gravitational action with induced non-minimal coupling of  $\phi$  to gravity (a similar mechanism was proposed in [11]):

$$\frac{M_p^2}{2} \int d^4x \sqrt{-g} R \rightarrow \frac{M_p^2}{2} \int d^4x \sqrt{-g} R - \frac{\xi}{2} \int d^4x \sqrt{-g} \phi^2 R . \quad (4)$$

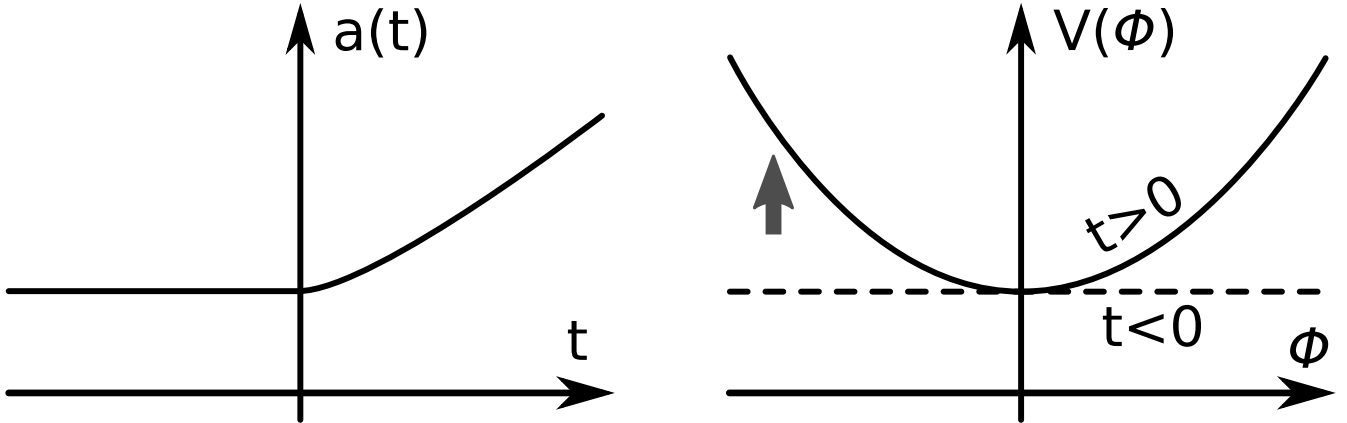


FIG. 1: An emergent potential from emergent cosmology.

This modified action also leads to a direct linear coupling between fluctuations of  $\phi$  and curvature perturbations.

For definiteness, in the present work, we only consider the first possibility, which is inspired by a Kähler potential in supersymmetry<sup>1</sup>. We expect that the modified gravity case should have a similar effect, via a conformal transformation.

### III. GALILEAN GENESIS EXAMPLE

#### A. Preliminaries

Consider the Galilean scenario [1]. In the Galilean case, the gravity sector is standard, with

$$S_g = \frac{M_p^2}{2} \int d^4x \sqrt{-g} R. \quad (5)$$

The Galilean field  $G$  has an action

$$S_G = \int d^4x \sqrt{-g} \left[ f^2 e^{2G} (\partial G)^2 + \frac{f^3}{\Lambda^3} (\partial G)^2 \nabla^2 G + \frac{f^3}{2\Lambda^3} (\partial G)^4 \right], \quad (6)$$

The Galilean-gravitation system has a solution describing an emergent universe:

$$G \simeq -\log(-H_0 t) - \frac{f^2}{2M_p^2 H_0^2 t^2}, \quad \rho_G \simeq \frac{f^4}{3M_p^2 H_0^4 t^6}, \quad p_G \simeq -\frac{2f^2}{H_0^2 t^4}, \quad H \simeq -\frac{1}{3} \frac{f^2}{M_p^2 H_0^2 t^3}, \quad (7)$$

where  $H_0 \equiv \sqrt{2\Lambda^3/(3f)}$  is a constant. This solution is valid at early times

$$t^2 \gg \frac{f^2}{M_p^2 H_0^2}. \quad (8)$$

Here we have solved the equation of motion of  $G$  to second order, and other equations to first order. As one can check, this suffices for the calculation of the equation of motion for the isocurvaton.

For simplicity, we assume that the Galilean field decays before condition (8) breaks down. Technically it should be straightforward to generalize our analysis to the  $t^2 \leq f^2/(M_p^2 H_0^2)$  region, provided that a well defined UV completion is given and the equations of motion are solved numerically.

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<sup>1</sup> In the case of a Kähler potential, it is more natural to have a standard kinetic term, leaving the exponential  $\exp\left[\mathcal{O}(1)\frac{\phi^2}{M_p^2}\right]$  coupling to the potential term. However, here we consider an over-all exponential term. For the Galilean case, we have to because there is no potential for the Galilean field. On the other hand, in the pseudo-conformal case, we could consider the case where the exponential only couples to the potential, and the result should not change much.

As noticed in [1], the  $G$  field has a blue spectrum, instead of a nearly scale invariant spectrum of perturbations. However, it was also noticed that if a massless scalar field  $\phi$  is introduced which couples to the Galilean like

$$S_\phi = -\frac{1}{2} \int d^4x \sqrt{-g} e^{2G} (\partial\phi)^2, \quad (9)$$

then in the Galilean background the  $\phi$  field feels an effective metric with a conformal factor  $e^{G/2}$ . Inserting the Galilean solution  $e^{2G} \simeq 1/(H_0 t)^2$ , it follows that  $\phi$  obeys the same equation of motion like a minimally coupled scalar field in de Sitter space and thus obtains a scale invariant power spectrum if the fluctuations start out in their quantum vacuum state. As discussed in [13], the ghost condensate can decay to excitations of the  $\phi$  matter field via the coupling in (9), thus allowing for a graceful exit from the stage of Galileon domination and onset of the usual radiation phase of Standard Cosmology.

However, as noticed in [13], there is a problem to convert the perturbation in  $\phi$  to curvature perturbations. This is because  $\phi$  has no potential or background motion. The curvature perturbation couples to the energy density of  $\phi$ , and is thus proportional to  $\delta\phi^2$ . As a result, the curvature perturbation is neither scale invariant nor Gaussian. If one tries to apply the curvaton scenario [17] and gives the  $\phi$  field a potential which leads to slow rolling, one encounters the next obstacle: now  $\phi$  has a non-vanishing background energy density which renders the emergent Galilean phase unstable.

Our main point is that if  $\phi$  is an originally massless direction, whose mass is uplifted by the Hubble parameter, then there is no such instability since the mass vanishes deep in the Galileon phase. Here we consider an example realizing this uplifting.

### B. An additional scalar with Kähler type coupling

In this subsection, we introduce a model where the additional scalar field arises from the Kähler potential, or a phenomenological analog. We consider the action

$$S_{eG} = \int d^4x \sqrt{-g} \left( 1 + \frac{\phi^2}{M^2} \right) \left\{ e^{2G} \left[ f^2 (\partial G)^2 - \frac{1}{2} (\partial\phi)^2 \right] + \frac{f^3}{\Lambda^3} (\partial G)^2 \nabla^2 G + \frac{f^3}{2\Lambda^3} (\partial G)^4 \right\}, \quad (10)$$

where  $M$  is a constant with the dimension of mass (could be real or purely imaginary). The gravitational sector is standard.

To simplify the calculation, we assume

$$t^2 \gg \frac{f^2}{M_p^2 H_0^2}, \quad \left| \frac{\phi^2}{M^2} \right| \ll 1. \quad (11)$$

The first inequality is discussed in Section III and is an early time approximation. The second inequality is a condition for the effective field theory expansion of the new physics to be valid, and it also ensures that  $\phi$  remains a sub-dominant component, i.e. a spectator scalar field.

At the homogeneous and isotropic background level, the matter action for  $\phi$  takes the form

$$S_e = \int d^4x \frac{1}{H_0^2 t^2} \left[ \frac{1}{2} \dot{\phi}^2 - \frac{f^4}{M_p^2 M^2 H_0^2 t^4} \phi^2 \right]. \quad (12)$$

To leading order in the approximations (11), we obtain the solution

$$\phi(t) \simeq \phi_0 - \frac{f^4 \phi_0}{5 H_0^2 M^2 M_p^2 t^2}, \quad (13)$$

where  $\phi_0$  is a constant. The second term is much smaller than the first one in a vast parameter space, considering that  $t^2$  is huge at early times. Thus the motion of  $\phi$  is very slow.

In the following calculation, we self-consistently treat  $\phi$  as a constant. We shall come back to the time variation of  $\phi$  when discussing the spectral index.

### C. Cosmic perturbations

We start from the ADM metric <sup>2</sup>:

$$ds^2 = -N^2 dt^2 + a^2 e^{2\psi} \delta_{ij} (N^i dt + dx^i)(N^j dt + dx^j) , \quad (14)$$

with perturbations  $\alpha$  and  $\beta$  defined as

$$N = 1 + \alpha , \quad N_i = \partial_i \beta . \quad (15)$$

The matter fields  $\phi$  and  $G$  are perturbed as

$$\phi \rightarrow \phi + \delta\phi , \quad G \rightarrow G + \delta G . \quad (16)$$

We have the freedom to set another gauge condition. However for the moment we shall not do that and leave the time shift  $t \rightarrow t + \delta t$  as a residual gauge symmetry.

With the assumptions in equation (11) <sup>3</sup>, one can solve for  $\alpha$  and  $\beta$  using the constraint equations and reinsert the solutions into the action. The second order Lagrangian then takes the form

$$\begin{aligned} \mathcal{L}_2 = & \left( \frac{3H_0 M_p^2 t}{f} \right)^2 \left( \dot{\psi}^2 - k^2 \psi^2 \right) \\ & + \left( \frac{f}{H_0 t} \right)^2 \left( \delta \dot{G}^2 - k^2 \delta G - \frac{2}{t^2} \delta G^2 \right) \\ & + \left( \frac{1}{H_0 t} \right)^2 \left( \frac{1}{2} \delta \dot{\phi}^2 - \frac{k^2}{2} \delta \phi^2 \right) \\ & + \left( \frac{3H_0 M_p^2 t}{f} \right) \left( \frac{f}{H_0 t} \right) \left( -2\dot{\psi} \delta \dot{G} + \frac{4\delta G \dot{\psi}}{t} + 2k^2 \psi \delta G \right) \\ & + \left( \frac{3H_0 M_p^2 t}{f} \right) \left( \frac{1}{H_0 t} \right) \left( \frac{4f\phi_0}{3M^2} \right) \left( -\dot{\psi} \delta \phi + 3\delta \phi \dot{\psi} + k^2 \psi \delta \phi \right) \\ & + \left( \frac{f}{H_0 t} \right) \left( \frac{1}{H_0 t} \right) \left( \frac{4f\phi_0}{3M^2} \right) \left( \delta \phi \delta \dot{G} - \delta \phi \dot{\delta G} - k^2 \delta \phi \delta G \right) \\ & - \left( \frac{3H_0 M_p^2 t}{f} \right)^2 \left( \frac{3f^4}{2M_p^4 H_0^4 t^6} \right) \psi^2 - \left( \frac{3H_0 M_p^2 t}{f} \right) \left( \frac{f}{H_0 t} \right) \left( \frac{2f^2}{H_0^2 M_p^2 t^4} \right) \psi \delta G , \end{aligned} \quad (17)$$

Here, the first three lines represent the free Lagrangian, and the 4th to 6th lines are the mixing terms. The coefficients of the two terms in the last line are small. We can self-consistently neglect these two terms. The self-consistency check involves obtaining the solution making the approximation, inserting back into the action, and checking that the contributions of these two terms are indeed small.

One can define canonically normalized fields via

$$\sigma \equiv \frac{3H_0 M_p^2 t}{f} \psi - \frac{f}{H_0 t} \delta G , \quad \chi \equiv \frac{1}{H_0 t} \delta \phi . \quad (18)$$

In terms of these fields the Lagrangian becomes

$$\mathcal{L}_2 = (\dot{\sigma}^2 - k^2 \sigma^2) + \left( \frac{1}{2} \dot{\chi}^2 - \frac{k^2}{2} \chi^2 + \frac{\chi^2}{t^2} \right) - \frac{4f\phi_0}{3M^2} \left( \frac{\dot{\chi}}{t} - \frac{2}{t^2} \chi \right) \sigma . \quad (19)$$

Note that  $\psi$  or  $\delta G$  appear in the above action only in terms of the gauge-invariant combination  $\sigma$ . On the other hand,  $\chi$  appears in a gauge-dependent form. This gauge-dependence is due to the approximation  $|\phi_0^2/M^2| \ll 1$  which we

<sup>2</sup> We have chosen a gauge in which the spatial metric is flat up to a conformal factor

<sup>3</sup> This is to say, neglecting terms which are small comparing to other existing terms in the two expansion parameters.

have made. By this approximation, we are asserting that  $\phi_0$  is sub-dominant, thus couples to gravity only weakly and is treated as a spectator scalar field on a fixed background. At higher orders, we would recover an explicitly gauge-invariant form. Note that if we were to choose a uniform  $\phi$  gauge, then the parameters in the gauge transformation between the gauge we are using and the uniform  $\phi$  gauge will be large, which mixes different orders of the  $\phi_0^2/M^2$  terms in the second order action.

To put it in another way, the  $|\phi_0^2/M^2| \ll 1$  approximation has separated gauge transformations into two classes, the “small gauge transformations” and “large gauge transformations”, as illustrated in Figure 2. Small gauge transformations transform between uniform total energy density slice and the flat slice, or any other slices not too far away from it. On the other hand, large gauge transformations transform between the above slices and the uniform  $\phi$  gauge. The action (19) can be used when only small gauge transformations are considered. While to consider a large gauge transformation, one need to use the full second order action, without the approximation  $|\phi_0^2/M^2| \ll 1$ .

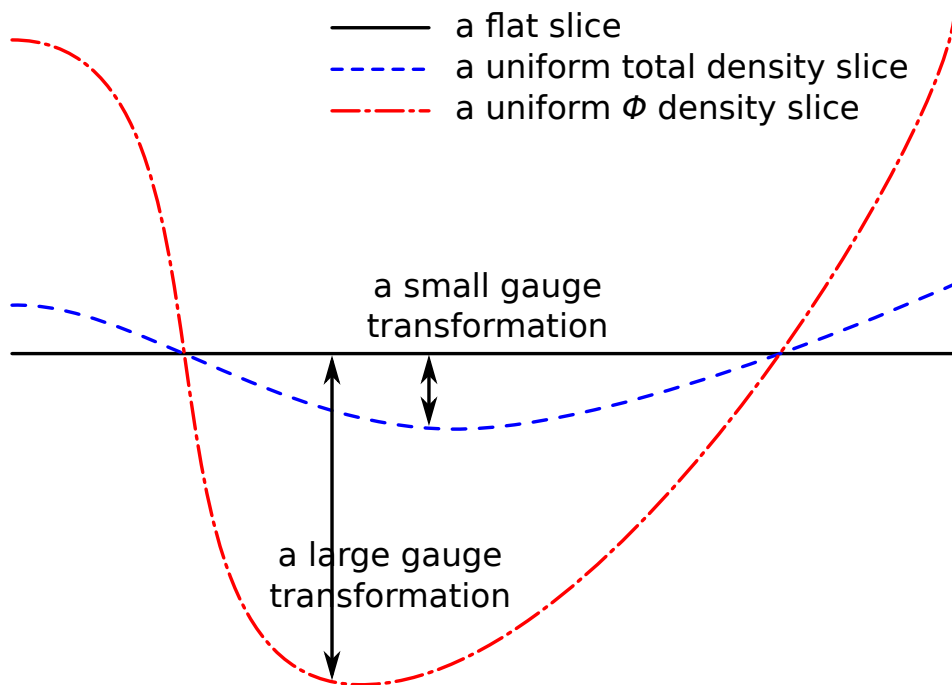


FIG. 2: Different slices, and “small” and “large” gauge transformations between them.

Now we make the further assumption that

$$\left| \frac{4f\phi_0}{3M^2} \right| \ll 1 \quad (20)$$

such that the last term in equation (19) can be treated as a perturbation when solving the equation of motion of  $\chi$ . Then the  $\sigma$  field has a free equation of motion

$$\ddot{\chi} + k^2\chi - \frac{2}{t^2}\chi = 0. \quad (21)$$

This is the same equation of motion as a free scalar field  $\chi$  in a de-Sitter background (with time  $t$  being replaced by the conformal time). This demonstrates that  $\chi$  will acquire a scale-invariant spectrum in this background [1] if it starts out in its vacuum state.

The back-reaction of  $\chi$  on  $\sigma$  is given by

$$\ddot{\sigma} + k^2\sigma + \frac{2f\phi_0}{3M^2} \left( \frac{\dot{\chi}}{t} - \frac{2\chi}{t^2} \right) = 0. \quad (22)$$

In the context of the emergent paradigm it is natural to assume that both canonical fields start in their vacuum state. This is similar to the assumption that scalar matter fields in the de Sitter phase of inflationary cosmology begin in

their vacuum state - although the justification of these assumptions are very different. With these initial conditions, the free evolution for  $\chi$  leads to the following form of the mode functions:

$$\chi = \frac{e^{-ikt}}{\sqrt{2k^3}} \left( \frac{1}{t} + ik \right). \quad (23)$$

Inserting this solution into the  $\sigma$  equation, we have

$$\sigma = \frac{e^{-i(k t + \theta_k)}}{\sqrt{k}} + \frac{e^{-ikt} f \phi_0}{\sqrt{2k^3} M^2 t} + \frac{ie^{ikt} f \phi_0}{3\sqrt{2k} M^2} \text{Ei}(-2ikt) - \frac{ie^{-ikt} f \phi_0}{3\sqrt{2k} M^2} \log(kt), \quad (24)$$

where Ei stands for the exponential integral function. Here, the first term comes from the quantum fluctuation of  $\sigma$ , and the rest of the terms are sourced by  $\chi$  fluctuations. And  $\theta_k$  is a relative phase between the original quantum perturbations of the Galileon and the induced perturbations. Note that the last two terms are subdominant because on super-Hubble scales  $-kt \ll 1$ , and thus they are small compared to the second term. For the same reason, the second term will dominate over the first term in most of the parameter space.

As discussed above, we can safely work in the uniform total energy density slice, in which the curvature perturbation is  $\zeta = \psi$ . Making use of (18) and (24) and inserting into the definition of the power spectrum we find the scale-invariant result

$$P_\zeta(k) = \frac{f^4 \phi^2}{36\pi^2 H_{\text{eff}}^2 M_p^4 M^4 t^4}, \quad (25)$$

where  $H_{\text{eff}} \simeq H_0 + f^2/(2M_p^2 H_0 t^2)$  is the effective Hubble parameter at Hubble crossing, which can be inferred from the second order solution of the equation of motion of  $G$ <sup>4</sup>. This result is true for all times  $t$  before the time  $t_D$  when either the Galileon or the field  $\phi$  decays. For simplicity, we shall assume that both fields decay into normal radiation at the same time  $t_D$  (e.g. via the process studied in [13]). Then, the time  $t_D$  is the time when the conversion of isocurvature perturbation to curvature fluctuation shuts off. Note that it is the non-trivial coupling between  $\phi$  and  $\chi$  introduced in our construction which leads to the scale-invariance of the curvature fluctuations. The intrinsic spectrum of curvature fluctuations (in the absence of a  $\phi$  field) would be deep blue (scalar spectral index  $n_s = 3$ ), as can be verified by doing the calculation with the first term in (24) only.

The power spectrum is growing until the  $G$  and  $\phi$  fields decay. To relate to observations, the time appearing in the power spectrum should be taken to be the time  $t_D$  when the Galileon and spectator scalar field have decayed into radiation. After the time  $t_D$ , the isocurvature mode disappears and the curvature fluctuation remains constant in time (for modes of interest to us which are far outside the Hubble radius in the radiation phase which starts after the time  $t_D$ ).

Note that the increase of the power spectrum in time does not spoil the scale invariance because the scale invariance is determined by the  $k$ -dependence of the power spectrum. Instead, the increase in time indicates that the perturbation in the isocurvature direction is continuously converting to curvature perturbation before it decays. This time dependence is similar to what occurs in the curvaton scenario in inflationary cosmology where the presence of isocurvature fluctuations see a continuously growing curvature mode<sup>5</sup>.

The amplitude of the resulting spectrum of curvature fluctuations depends on the values of the mass scales in the Lagrangian, and on the values of  $\phi_0$  and  $t_D$ . It is natural to assume  $f \sim M \sim M_p$ , and to choose  $\Lambda$  to be parametrically larger than  $f$ , i.e.  $\Lambda = \alpha f$  with the constant  $\alpha \gg 1$ . For consistency of the approximations, we must choose  $\phi_0$  parametrically smaller than  $M$ , i.e.  $\phi_0 = \beta M$  with constant  $\beta \ll 1$ . In this case, we find

$$P_\zeta \sim \beta^2 \alpha^{-3} (M_p t_D)^{-4} \quad (26)$$

<sup>4</sup> From the second order action, we can only determine the leading order result with  $H_{\text{eff}} \simeq H_0$ . However, we can solve the relation between  $H_{\text{eff}}$  and  $H_0$  to higher orders using the background field equation  $G \simeq -\log(-H_0 t) - \frac{f^2}{2M_p^2 H_0^2 t^2}$ . This second order solution is needed if we want to calculate the spectral index. Similarly, we replaced  $\phi_0$  to  $\phi$  in the power spectrum to take into account the slow but non-vanishing  $k$ -dependence.

<sup>5</sup> In the curvaton scenario, where the curvaton  $\rho_c$  and the inflaton are decoupled (except via gravity), there is a conserved quantity  $\zeta_c = -\psi - H\delta\rho_c/\dot{\rho}_c$ , where  $\rho_c$  is the energy density of the curvaton field. The total curvature fluctuation  $\zeta$  is not conserved before the decay of curvaton. Instead, it is related to  $\zeta_c$  by  $\zeta = \zeta_c \dot{\rho}_c/\dot{\rho} = \zeta_c(\rho_c + p_c)/(\rho + p)$ , where  $\rho$  and  $p$  are the total energy and total pressure. In our case,  $\rho + p \propto t^{-4}$  and  $\rho_\phi + p_\phi \propto t^{-6}$  (which is the analog of  $\rho_c + p_c$ ). Thus  $\zeta$  scales as  $t^{-2}$  and  $P_\zeta$  scales as  $t^{-4}$ . However, there is explicit coupling between  $\phi$  and the Galileon. Thus while we hope this footnote still provides an intuitive understanding, an explicit calculation is also needed, as we give in the main text.

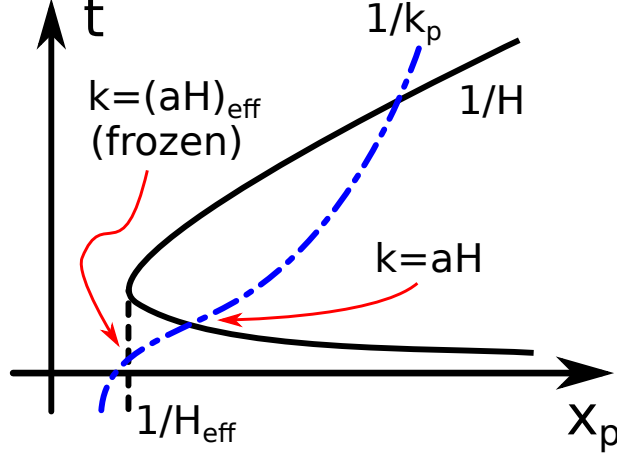


FIG. 3: The effective Hubble crossing time should be calculated as  $k = a_{\text{eff}}H_{\text{eff}}$  instead of  $k = aH$  because the former is the time when the fluctuations freeze out, while the latter plays no special role in the equation of motion for the fluctuations.

which is parametrically smaller than 1 if  $|t_D M_p| \sim 1$ .

To calculate the spectral index, we need to compare different  $k$  modes. As usual, the difference in  $k$  is translated into a difference in the time  $t(k)$  when the mode  $k$  crosses the effective Hubble radius in the emergent phase. The reason why we must use the effective Hubble radius crossing  $k = a_{\text{eff}}(t_k)H_{\text{eff}}(t_k)$  and not the crossing of the background Hubble radius  $k = aH$  is that it is the effective Hubble radius which determines the transition from the phase when fluctuations oscillate and when they are frozen out (see Figure 3 for an illustration of the scales involved).

The spectral index can be calculated as

$$n_s - 1 = \frac{d \ln P_\zeta}{d \ln k} = -t \frac{d \ln P_\zeta}{dt} = -2t \frac{\dot{\phi}}{\phi} + 2t \frac{\dot{H}_{\text{eff}}}{H_{\text{eff}}}, \quad (27)$$

Inserting the equations of motion, we have

$$n_s - 1 = - \left( \frac{2f^2}{5M^2} + 1 \right) \frac{2f^2}{M_p^2 H_0^2 t_k^2}. \quad (28)$$

The spectral index could be extremely small. This is because we want to generate a large hierarchy of scales for which the spectrum is scale invariant. Let the number of e-foldings of the effective de Sitter phase be  $N$ . Then we need

$$\frac{t_k}{t_D} = \frac{a_{\text{eff}}(t_D)H_{\text{eff}}(t_D)}{a_{\text{eff}}(t_k)H_{\text{eff}}(t_k)} = e^N. \quad (29)$$

Thus, the spectral index is very small given a large  $N$ . This can be verified by inserting the above expression into (28). We get

$$\alpha^{3/2} M_p t_D \simeq \frac{1}{e^N \sqrt{1 - n_s}}. \quad (30)$$

Present experiments suggest that  $1 - n_s$  is a few percent. In principle, the free parameters  $\alpha$ ,  $\beta$  and  $t_D$  can be chosen such that the power spectrum amplitude (26) agrees with observations and the tilt reproduces the best-fit value (30). However, it may be difficult to fit the current data of  $n_s$  because the presence of the exponential factor  $e^N$ . Additional mechanisms may be needed to generate a tilt of the power spectrum.

#### IV. PSEUDO-CONFORMAL MODEL

Similarly, for the pseudo-conformal model of [12], we consider the matter action

$$S_m = \int d^4x \sqrt{-g} \left( 1 + \frac{\phi^2}{M_1^2} \right) \left[ -\frac{1}{2}(\partial C)^2 + \frac{\lambda}{4}C^4 - \frac{C^2}{2M_2^2}(\partial\phi)^2 \right], \quad (31)$$



where  $C$  is the pseudo-conformal field, which drives the background motion of the emergent cosmology. The factor  $(1 + \phi^2/M_1^2)$  is again the new factor which we are adding, a factor which is motivated by the Kähler potential or its phenomenological analog.

At the background level, the dynamics of the scale factor, and of the fields  $C$  and  $\phi$  goes as

$$H = \frac{1}{3\lambda t^3 M_p^2}, \quad C = \sqrt{\frac{2}{\lambda}} \frac{1}{t}, \quad \phi = \phi_0 t^{-\frac{2M_2^2}{3M_1^2}}. \quad (32)$$

One can set up cosmic perturbations following what was done in the previous section. Self-consistently, we postulate that the following quantities are small

$$1/(6\lambda t^2 M_p^2), \quad |\phi^2/M_1^2|, \quad |M_2\phi_0/M_1^2|, \quad |M_2^2/M_1^2|. \quad (33)$$

The first three quantities are analogs of conditions imposed in the Galilean Genesis example, corresponding to early time, sub-domination of  $\phi$ , and weak coupling of fluctuations between the two degrees of freedom, respectively. The requirement of the smallness of the 4th quantity is new. It comes about as follows: If  $|M_2^2/M_1^2| \geq 1$ , then the  $\phi$  field will roll down its emergent potential too quickly, and no scale invariant spectrum will be generated.

With the above assumptions, the “small” gauge invariant quantities for fluctuations can be constructed as

$$\sigma \equiv 3M_p^2 t \sqrt{\lambda} \psi - \frac{\delta C}{\sqrt{2}}, \quad \chi \equiv \frac{\sqrt{2} \delta \phi}{M_2 t \sqrt{\lambda}}. \quad (34)$$

Using these definitions, the second order Lagrangian can be written as

$$\mathcal{L}_2 = (\dot{\sigma}^2 - k^2 \sigma^2) + \left( \frac{1}{2} \dot{\chi}^2 - \frac{k^2}{2} \chi^2 + \frac{\chi^2}{t^2} \right) + \frac{2\sqrt{2}M_2\phi_0}{3M_1^2} \left( \frac{\dot{\chi}}{t} - \frac{2}{t^2} \chi \right) \sigma. \quad (35)$$

Interestingly, the pseudo-conformal case and the Galilean case share the same structure of the perturbations, except for a different sign in the interaction term, which is only a different convention and thus unimportant<sup>6</sup>.

Thus we have the equations of motion in the Born approximation:

$$\ddot{\chi} + k^2 \chi - \frac{2}{t^2} \chi = 0, \quad (36)$$

$$\ddot{\sigma} + k^2 \sigma - \frac{2\sqrt{2}M_2\phi_0}{3M_1^2} \left( \frac{\dot{\chi}}{t} - \frac{2\chi}{t^2} \right) = 0. \quad (37)$$

Most naturally, the fields should start from a vacuum state. Thus

$$\chi = \frac{e^{-ikt}}{\sqrt{2k^3}} \left( \frac{1}{t} + ik \right). \quad (38)$$

Inserting the solution into the  $\sigma$  equation, we have

$$\sigma = \frac{e^{-i(k t + \theta_k)}}{\sqrt{k}} - \frac{e^{-ikt} M_2 \phi_0}{\sqrt{k^3 M_1^2 t}} - \frac{i e^{ikt} M_2 \phi_0}{3\sqrt{k} M_1^2} \text{Ei}(-2ikt) + \frac{i e^{-ikt} M_2 \phi_0}{3\sqrt{k} M_1^2} \log(kt), \quad (39)$$

where  $\theta_k$  is a relative phase between the original quantum perturbations of  $C$  and the induced perturbations.

As in the previous section, the second term dominates the  $-kt \ll 1$  regime. Thus, the power spectrum is

$$P_\zeta = \frac{M_2^2 \phi_0^2}{18\pi^2 \lambda M_p^4 M_1^4 t_D^4}, \quad (40)$$

where  $t_D$  is the time when the pseudo-conformal phase ends and the isocurvature field decays. Again, the spectral index is small and the spectrum is nearly scale invariant.

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<sup>6</sup> Technically, there are actually two differences between the calculations in the Galilean case and in the pseudo-conformal case. First, in the pseudo-conformal example, although the time evolution of  $\phi$  is still slow, it is now of the same order as the interaction term in the second order Lagrangian (the third term in Equation (35)). Thus the time evolution of  $\phi$  cannot be neglected. In the Galilean case,  $\phi$  can be treated as a constant. Second, in deriving the interacting term in Equation (35), we have used the free field equation of motion for  $\chi$ . Strictly speaking, we are not allowed to use the equation of motion in deriving the action. However, here the use of the equation of motion should be understood as a redefinition of the perturbation variables, as in the case of the field redefinition done in [18].

## V. CONCLUSIONS

We have shown how correction terms to the effective action of the low energy fields which are expected based on supergravity or quantum gravity considerations induce a linear coupling between the fluctuations of an isocurvature field and the curvature fluctuations. Thus, a scale-invariant spectrum in a spectator scalar field induces a scale-invariant curvature fluctuation spectrum. Scale-invariant spectra for spectator scalar fields are induced e.g. in the Galileon Genesis model [1], in the Conformal Cosmology of [8] and in the Pseudo-Conformal scenario of [12]. Our mechanism can be used to demonstrate that these scenarios indeed lead to a scale-invariant spectrum of curvature perturbations.

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